MEASUREMENT OF STAR DIAMETERS BY THE INTERFEROMETER METHOD.

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The idea of measuring the angular diameter of a fixed star by the method of interference of light beams was suggested by Fizeau in his report on the Bordin Prize to the French National Academy in 1868. Stephan spent the year 1874 examining all the brighter stars by this method with the 31.5-inch telescope of the Marseilles Observatory, and rightly found that the telescope was altogether too small for the purpose. The papers relating to this work will be found in *Comptes Rendus,* LXVI., p. 1008, 1873, and LXVIII., p. 1008, 1874.

Nothing further appeared on this subject until Michelson in 1890 published a masterly paper in the *Philosophical Magazine,* "On the Application of Interference Methods to Astronomical Measures." He pointed out that the telescope itself need not be large, and that the results desired might be accomplished by the use of a double periscopic arrangement consisting of four small auxiliary mirrors placed in a frame on the end of the telescope. Designs for a periscopic attachment are shown by Michelson in his paper but his successful measures of the satellites of Jupiter with the 12-inch refractor of the Lick Observatory in 1891 were made with two apertures placed directly in front of the objective. Aside from this question of size, the interferometer was considered an instrument of the utmost precision requiring the best of optical conditions, and the belief was general that the disturbances of the atmosphere would probably be such as to prevent its successful use even if a large telescope should be built. After the 100-inch Hooker telescope had been completed and excellent results obtained with it at the full aperture of the mirror, Director Hale invited Dr. Michelson to investigate the question of interference with this large instrument. Stephan had already found fringes with apertures separated by 25.6 inches; Michelson found the same result in turn, first with the full aperture of the 40-inch Yerkes Refractor, then with the 60-inch reflector at Mount Wilson and finally the 100-inch Hooker Reflector, even though the seeing was not very good. The immediate result was the application of interference methods to the measurement of double stars, and Anderson developed a method by means of which he determined the distance between the components of Capella with a very great degree of precision, it being impossible to do this by ordinary methods on account of the closeness of the companions to one another. The next step undertaken was to adapt the great reflector in accordance with Michelson's plan of 1890, by mounting four auxiliary small mirrors on a beam placed across the upper end of the telescope tube. Success attended its installation and in August, 1920, the interference fringes obtained on Vega with an aperture equivalent to 18 feet were as easily seen as those at 6 feet.

Meanwhile, Eddington, Russell, Shapley, and others had made calculations of the diameters of a number of stars based on estimates of their surface brightness, and their results indicated that α Orionis would be an excellent object for an attempt to measure the diameter. Merrill first examined the star with the apparatus used by Anderson in the measurement of Capella and found a definite decrease in the visibility of the interference fringes with the slits separated the full aperture of the mirror. An actual measurement of the diameter of α Orionis was then made by the writer on Dec. 13, 1920; a description of the instrument and method is given below.

Before describing the 20-foot interferometer in detail it may be well to recall a few of the principles of interferometry. Thomas Young found that two pencils of light from a point source, when brought together again, can be made to produce interference bands or "fringes." A pinhole in a screen A (Fig. 1) in front of a candle will, for laboratory experiments, serve as a source of light. Light spreads from this pinhole in concentric spherical waves and is intercepted by a screen B, having in it two pinholes equidistant from the axis so that $AC = AD$. These openings furnish the two pencils of light and interference takes place where the wave fronts intersect, say at a screen E. Having left B at the same instant, the two pencils arrive at the screen E on the axis at the same time, the crest or trough of one wave falling upon the corresponding crest or

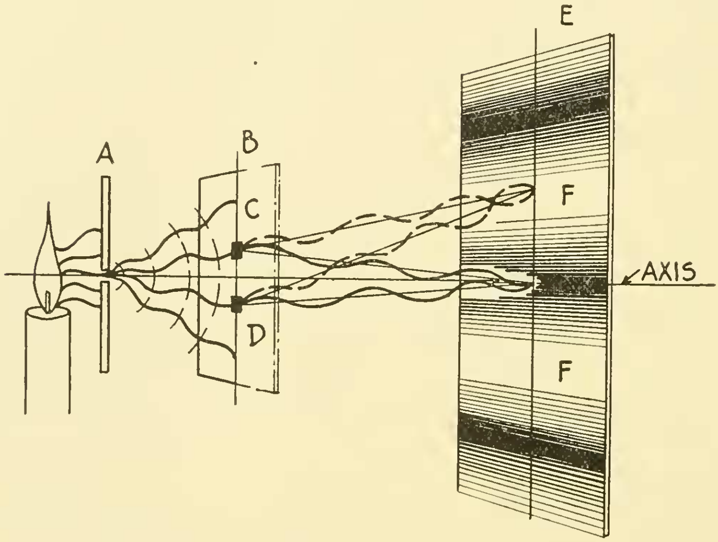

FIG. 1. Interference of two light pencils.

trough of the other wave, each thus reinfocing the other. At F, F are points on either side of the axis where the crest from one wave falls in the trough of the other; the result is the complete neutralization of the one by the other (and the total absence of motion) and consequently darkness at that point. Between these two points there is a gradual change in intensity, so that one grades into the other. The observed result is a series of parallel bands, alternately bright and dark, lying on both sides of the axis, bright wherever the path difference from the screen E to the two apertures is equal to any number of even half wave-lengths of light as 2, 4, 6, etc., and dark when the number of half waves in the path difference is an odd number as 1, 3, 5. No light is lost in the mutual action of the two pencils on one another; it is simply redistributed, and what is removed from the dark region is to be found in the brighter portion.

Fresnel improved the method of observing the bands by setting two prisms of equal angle side by side as indicated in Fig. 2. Light from a source *G* passing through the two prisms arrives at a screen *H*, and here the same principle of crest on crest or crest on trough produces interference fringes. It should be borne in

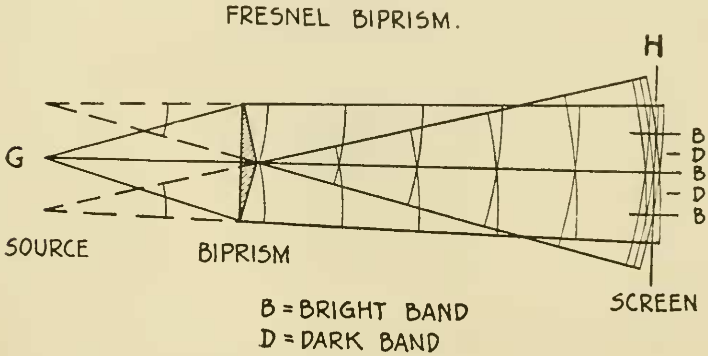

FIG. 2. Fresnel biprism.

mind that light from two separate sources cannot interfere; it is only two pencils of light from the same source that produce the phenomenon in question. As the size of the source of light is slightly increased, each point in it produces a series of concentric spherical waves and there are therefore at the screen a great many overlapping interference fringe patterns slightly displaced with respect to one another, which reduce the contrast between the dark and light bands. Without going into detail, it is found mathematically and experimentally that for a given distance between the slits there is a certain size of source for which this overlapping is complete, fringes are not visible and ordinary illumination takes place. As the source is slightly increased in size the fringes reappear much less conspicuous than before and then vanish again.

In the application of this principle to the telescope, the wave front from a distant star is considered a plane although it is actually a portion of a sphere. Each point of the star produces a wave which is inclined slightly to that from the central point. All these waves superimpose in apparently one wave front on the axis of the telescope and for a distance of a foot or two on either side. When the telescope is covered with a screen having two apertures fairly close together there appears at the focus of the telsecope a series of interference fringes very sharp and clear cut, superposed on the ordinary diffraction image (Fig. 3) of the star; this image

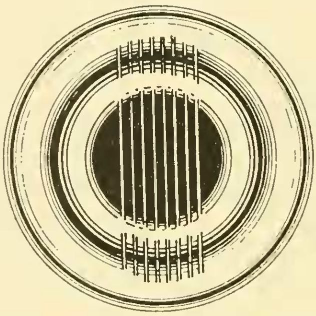

Fig. 3. Diagram of star image showing diffraction rings and interference fringes.

is present at all times, even though the fringes may not be visible. It consists of a central bright disk surrounded by alternate bright and dark rings, the relative brightness of which depends upon the size of the openings. In all cases, however, the outer rings are fainter than the central disk.

As the distance between the slits in front of the telescope is increased, the portions of the wave fronts from any two points of the source passing through them are, owing to their mutual inclinations, further apart and no longer in phase. Consequently overlapping of patterns takes place at the focus and a diminution in the relative contrasts of the fringes ensues. As the slits in front of the telescope continue to be separated, a distance is reached where the fringes vanish altogether. It has been found that the angular diameter of an artificial star disk, uniformly illuminated, is equal to $1.22\ \lambda/b$, where λ is the wave-length of the light used and b is the distance in question.

The full curve drawn in Fig. 4 represents the relationship between the visibility or clearness of the fringes and the relative separation of the slits for a given disk and color and it shows that the fringes are brightest when the openings are close together, that it decreases rapidly as the slits are separated and that it is zero when the ratio of λ/a is 1.22. As the slits are still further separated the fringes reappear and disappear several times, but are always increasingly weaker.

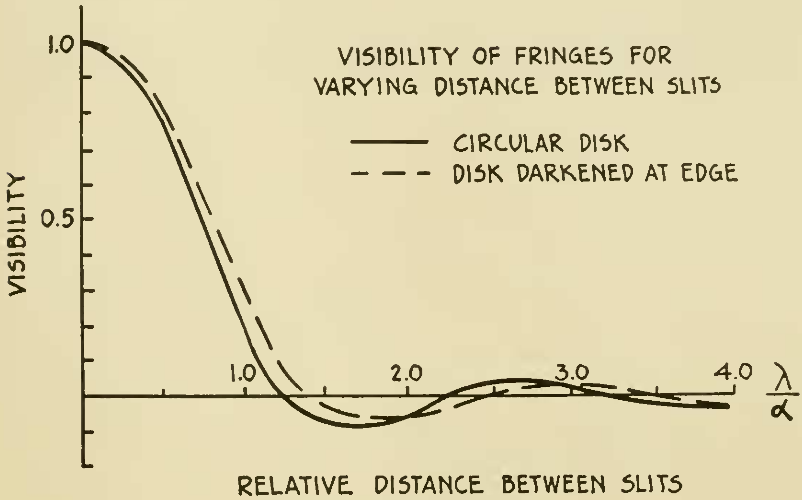

FIG. 4. Visibility curves.

It has been found for the sun that the brightness of its surface is not uniform and that it fades away towards the limb. The dotted line represents the manner in which its visibility varies as the distance between the slits is changed, and indicates that a factor of 1.33 is to be used, instead of 1.22, for a type of star similar to the sun.

There is no known star for which total disappearance of the fringes can be obtained with the aperture of the 100-inch reflector; α Orionis, as mentioned above, presented only a falling off of intensity at the extremities of the diameter of the mirror.

Accordingly, for the actual measurement of star diameters Michelson and Pease designed a beam (Fig. 5) to carry the four auxiliary mirrors, following Michelson's plan of 1890. This is placed upon the end of the Hooker telescope and the observations are made at the Cassegrain focus, which has an equivalent focal length of 134 feet. The pencils of light from the star are reflected from the two outer mirrors, which are adjustable, to two inner mirrors. They then follow the ordinary path in the telescope, first to the great mirror, thence to the convex mirror, finally to a flat, which for convenience of observation brings the pencils to the focus at the side of the instrument.

The bed of the cross-beam consists of two 10-inch steel channels 21 feet long with their flanges turned inwards, separated by sections

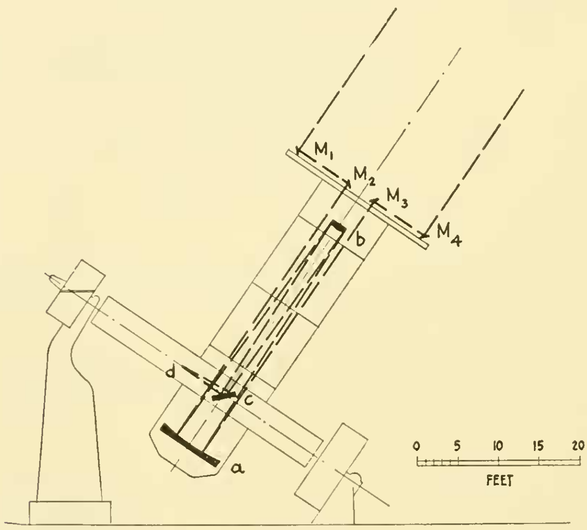

FIG. 5. Twenty-foot Interferometer Beam on 100-inch Reflector—Diagram of Light Paths.

of 12-inch channel and boxed on the lower side with 0.187-inch steel plate, all riveted securely together. Openings were cut whereever possible to decrease the weight and the inner edges of the top flanges were planed true to 0.001 inch, the frame being supported in the position in which it was to be mounted on the telescope.

The four mirrors are 6 inches in diameter, inclined at an angle of 45°, the outer ones facing upwards, the inner ones facing downwards; they are mounted on slides and the two outer ones, thus far moved by hand, are being equipped with screws driven by a single motor, to keep them equidistant from the inner mirrors.

Since the inner mirrors are fixed 45 inches apart, the spacing of the fringes at the focus is constant and equal to 0.008 inch. Experience has shown that fringes of this size can be conveniently examined with an eyepiece of one inch focus. After the beam is placed on the end of the telescope the inner mirrors are first adjusted in the daytime by mounting a miniature lamp at the focus and observing its position, after reflection through the entire mirror system, as seen projected in the outer mirror when viewed from above the latter. Adjustment is made until the lamp apparently lies central in the outer mirror. At night a bar 45 inches long is placed across the flat-mirror mounting near the focus, centrally across the axis and then by sighting upward, the outer mirrors are adjusted until the end of the marker is near the center of the small region over which the star can be seen reflected in the inner mirrors. Observation at the focus now shows the star image lying somewhere within a field 20 inches in diameter, and further adjustment of the outer mirrors places the two beams centrally in this opening and in coincidence with each other.

To obtain a reference or zero image, the entire end of the telescope is covered save for two 6-inch openings, in addition to those of the interferometer mirrors, and the two admitted pencils pass over the ordinary course in the telescope and form an image at its focus. Usually the interferometer image is brought within a quarter of an inch of this image and both are viewed in the eyepiece simultaneously.

To obtain interference fringes the optical distance from the star to the focus along the two pencils must be the same. Experience has shown that it is very difficult to obtain this equality by simple motion of one of the mirrors along the beam. Compensation is accomplished by placing, about two feet within the focus, a double wedge of glass in the path of one pencil and a plane parallel plate of glass equal to the mean thickness of the wedges in the other path. The outer mirrors are first set equidistant from the inner mirrors by steel scales, then, observing at the eyepiece, one of the wedges is slowly shifted so as to increase or decrease the equivalent glass thickness until a point is reached at which the fringes appear. As light travels faster in air than in glass we must add glass if the path is relatively too long or take it out if too short. If glass thickness must be added in moving from the zero fringes to the reflector fringes, the path is too short; to equalize the wedge is backed off and the mirrors on this side of the interferometer beam separated by an amount indicated by the distance the wedge has been shifted. The angle of the prism is about 10 degrees and a linear motion of the prism of 1 mm. corresponds to a path difference of 0.09 millimeter. If glass be subtracted in moving from the zero fringes to the reflector fringes, the mirrors must move closer together in order that both sets of fringes may appear in the eyepiece at the same time. The wedge is shifted by means of a rod; one turn of this rod moves the wedge 0.5 mm. and compensates for 0.045 mm. of air path. Fringes can be observed throughout one-third of a turn of this rod, corresponding to a path difference of about 26 light waves.

Having in mind the operation of the interferometer and the appearances to be expected, we will turn to the observations made with the telescope in operation.

Several days were spent by the writer in November, 1920, in preparing the beam for operation, but as several important alterations were necessary, actual work was not begun until December. On December 13, with the outer mirrors at 10 feet separation the instrument was put in complete adjustment by observing β Persei and γ Orionis, both stars known to have diameters much smaller than can be measured with this instrument. This adjustment meant that, upon observing at the eyepiece, both the reference and the interferometer images were seen with fringes superimposed upon them. Upon turning to α Orionis the "zero" fringes were seen but no glimpse could be obtained of the interferometer fringes.

Turning to α Canis Minoris both sets of fringes were visible simultaneously, indicating that the instrument was in complete adjustment and that the disappearance of the fringes on α Orionis was real.

It may be thought that reliance cannot be placed upon a null measurement; there is no reason for this assumption as any instrumental flexure or atmospheric disturbance requires but a very slight adjustment of the wedge, and at this time the seeing was very good. Dr. Anderson was present on this night and checked the writer's observations. Further observation of α Orionis was not attempted on the succeeding nights in December because the seeing was poor, as was indicated by a reduction in the visibility of the zero fringes; consequently observations were made on α Ceti, α Tauri, and β Geminorum, with the outer mirrors at 13 feet separation. In February, with mirrors approximately 16 feet and 19 feet apart, observations were made on α Orionis, α Tauri, β Geminorum and α Bootis. The seeing did not warrant drawing any definite conclusions except that fringes were seen at all points for β Geminorum. This indicates that an interferometer with a base longer than 20 feet will be required to measure its diameter. Fringes were seen for α Tauri at 13 feet, at 19 feet, and in March at 14.5 feet, the visibility becoming less with increased separation of the mirrors. Additional measures will be made at points between 16 feet and 18 feet to see whether the fringes disappear as calculations indicate they should. For α Bootis the fringes were much reduced at 16 feet and could not be seen at 19 feet; the seeing was bad, however, and the observation indecisive.

Many stars have been used for checking the instrument; among them δ Tauri, γ Orionis, α Canis Minoris, α Geminorum, and η Bootis. All have shown strong fringes at 19 feet.

Experience has shown throughout that better seeing is required for this work than was at first supposed, particularly when the mirrors are widely separated and the visibility of the fringes is approaching the point where they disappear. Change of seeing at these times will cause the fringes to flicker in and out, but a check is always at hand, for at the same time the visibility of the "zero" fringes is also reduced.

On some occasions in bad seeing the zero fringes will remain fixed, but the interferometer fringes will shift to the side of the image, probably because small sections of the wave front become inclined to the general wave front, due to varying atmospheric densities.

Having determined the distance at which the fringes vanish, we find the angular diameter of the object from the expression $\alpha = 1.22\lambda/b$ where α is the angular diameter in radians (206265″), λ is the effective wave-length (in cm.) of the star or that portion of the spectrum which is most predominant in forming the fringes seen by the eye of the observer and b is the distance apart of the mirrors (in cm.); Anderson has found, in connection with his work on Capella, that the effective wave-length of a solar type star is

5.5×10^{-5} cm. and it is assumed for α Orionis that the value of the wave-length is 5.75×10^{-5} cm., a true value for which must be found by direct experimental work. The value of b found for α Orionis is 121 inches ($\pm$ 10 per cent.). The approximate value then for the angular diameter of α Orionis is .047″. The agreement of this value with those obtained by calculation, which range from .031″ to .051″, is striking. If there is a falling off of intensity toward the limb, as in the case of the sun, Michelson finds this value would be increased by 17 per cent. Several determinations of the parallax have been made for α Orionis and from these its distance b may be found from the expression

$$d = \frac{206265R}{\pi} \text{ miles,}$$

where $R = 93{,}000{,}000$ miles, the distance of the earth from the sun, and π is the value of the parallax in seconds of arc. Measures of the parallax thus far obtained are: Adams, .013″; Yale, .032″; Schlesinger, .016″; Yerkes, .022″, the weighted mean of which is .020″. From these values the distance is about 9.6×10^{14} miles. Knowing the distance and the angular value of the star, its linear diameter is found to be 218×10^{6} miles. This value is not a definite figure but only an approximation; but in any case it means that the diameter of the star is several times the distance between the earth and the sun and several hundred times the diameter of the sun itself.

The work is being continued until the half dozen stars which calculations indicate as measurable with the twenty-foot beam have been investigated. Most of these are stars having late type spectra. In order to measure diameters of early type stars such as Sirius and Procyon a much longer base is needed. For this work an interferometer with mirror separations as great as 50 or 100 feet has been discussed but it is felt the present instrument should be used to its limit and many data accumulated, particularly regarding seeing conditions with the mirrors widely separated, before anything definite is attempted in the way of a larger instrument.

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